MENIIT

NEET | IIT-JEE | FOUNDATION

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JEE MAIN-2020

COMPUTER BASED TEST (CBT)

DATE : 06-09-2020 (SHIFT-2) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

QUESTION & SOLUTIONS

PART-A : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

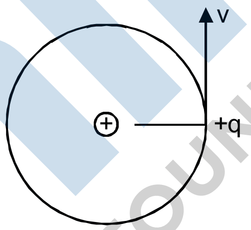
Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

1. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field $\vec{B}$. The magnetic moment of this moving particle is :

(1) $\frac{mv^2\vec{B}}{B^2}$ (2) $\frac{mv^2\vec{B}}{2B^2}$ (3) $-\frac{mv^2\vec{B}}{2B^2}$ (4) $\frac{mv^2\vec{B}}{2\pi B^2}$

Ans. (3)

Sol. $i = \frac{q}{T} = \frac{qv}{2\pi r}$



$M = i \times \pi r^2$

$= \frac{2v}{2\pi r} \times \pi r^2$

$M = \frac{1}{2} q.v.r$

$= \frac{1}{2} qv \times \frac{mv}{qB}$

$M = \frac{mv^2}{2B}$

Direction of $\vec{M}$ is opposite of $\vec{B}$ therefore

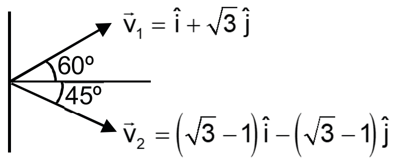
$\vec{M} = \frac{-mv^2\vec{B}}{2B^2}$

2. Particle A of mass m1 moving with velocity $\left(\sqrt{3}\,\hat{i} + \hat{j}\right)ms^{-1}$ collides with another particle B of mass m_2 which is at rest initially. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocities of particles A and B after collision respectively. $m_1 = 2m_2$ and after collision $\vec{v}_1 - \left(\hat{i} + \sqrt{3}\,\hat{j}\right)ms^{-1}$, the angle between $\vec{v}_1$ and $\vec{v}_2$ is :

(1) 105º (2) 15º (3) –45º (4) 60º

Ans. (1)

Sol. $m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$



$2m_2\left(\sqrt{3}\,\hat{i} + \hat{j}\right) + m_2 \times 0 = 2m_2\left(\hat{i} + \sqrt{3}\,\hat{j}\right) + m_2 \times \vec{v}_2$

$2\sqrt{3}\,\hat{i} + 2\hat{j} = 2\hat{i} + 2\sqrt{3}\,\hat{j} + \vec{v}_2$

$\vec{v}_2 = \left(\sqrt{3}-1\right)\hat{i} - \left(\sqrt{3}-1\right)\hat{j}$

$\vec{v}_1 = \hat{i} + \sqrt{3}\,\hat{j}$

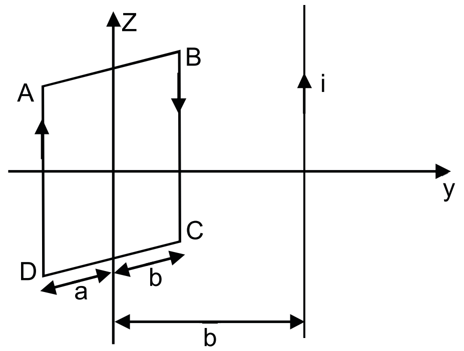
Angle between $\vec{v}_1$ and $\vec{v}_2$ is 105º

3. A square loop of side 2a and carrying current I is kept in xz plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point (0,b,0), (b > > a). The magnitude of torque on the loop about z-axis will be :

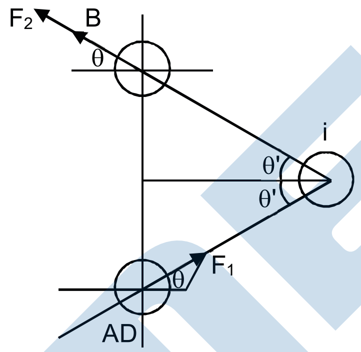
(1) $\frac{2\mu_0 I^2 a^2}{\pi b}$ (2) $\frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$ (3) $\frac{\mu_0 I^2 a^2}{2\pi(a^2+b^2)}$ (4) $\frac{\mu_0 I^2 a^2 b}{2\pi(a^2+b^2)}$

Ans. (2)

Sol.



Net force acting on wire AB & CD are zero



$\tau = F_1\cos\theta \times 2a = \frac{\mu_0}{2\pi}\frac{i^2}{\sqrt{a^2+b^2}}(2a) \times \frac{b}{\sqrt{a^2+b^2}} \times 2a$

$\tau = \frac{2\mu_0 i^2 a^2 b}{\pi(a^2+b^2)}$

4. Two identical electric point dipoles have dipole moments $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$ and are held on the x-axis at distance 'a' from each other. When released, they move along the x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is :

(1) $\frac{P}{a}\sqrt{\frac{3}{2\pi\epsilon_0 ma}}$ (2) $\frac{P}{a}\sqrt{\frac{1}{2\pi\epsilon_0 ma}}$ (3) $\frac{P}{a}\sqrt{\frac{1}{\pi\epsilon_0 ma}}$ (4) $\frac{P}{a}\sqrt{\frac{2}{\pi\epsilon_0 ma}}$

Ans. (2)

Sol.

$k_i + u_i = k_f + u_f$

$$0 - \frac{2k.p}{r^3} p_2 . \cos(180^\circ) = \frac{1}{2} mv^2 + \frac{1}{2} mv^2 + 0$$

$$mv^2 = \frac{2kp_1p_2}{r^3}$$

$$\Rightarrow \theta = \sqrt{\frac{2kp_1p_2}{mr^3}}$$

$p_1 = p_2 = p$ & $r = a$

$$v = \frac{p}{a}\sqrt{\frac{1}{2\pi \in_0 ma}}$$

5. For a plane electromagnetic wave, the magnetic field at a point x and time t is : $\vec{B}(x,t) = [1.2 \times 10^{-7} \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}]T$. The instantaneous electric field $\vec{E}$ corresponding $\vec{B}$ is :

(1) $\vec{E}.(x,t) = [36\sin(1 \times 10^3 x + 0.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$

(2) $\vec{E}.(x,t) = [36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{k}]\frac{V}{m}$

(3) $\vec{E}.(x,t) = [36\sin(1 \times 10^3 x + 1.5 \times 10^{11} t)\hat{i}]\frac{V}{m}$

(4) $\vec{E}.(x,t) = [-36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$

Ans. (4)

Sol. $E_0 = B_0 \times C = 1.2 \times 10^{-7} \times 3 \times 10^8 = 36$

As the light is propagating in x direction

& $(\hat{E} + \hat{B}) \,||\, \hat{C}$

$\therefore$ $\hat{E}$ should be in $\hat{j}$ direction

So electric field $\vec{E} = E_0 \sin \vec{E}.(x,t) = [-36\sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\hat{j}]\frac{V}{m}$

6. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion, is described by $y(t) = y_0 \sin^2 \omega t$, where 'y' is measured from the lower end of upstretched spring. Then ω is :

(1) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (2) $\sqrt{\frac{g}{y_0}}$ (3) $\sqrt{\frac{2g}{y_0}}$ (4) $\sqrt{\frac{g}{2y_0}}$

Ans. (4)

Sol. $y = y_0 \sin^2 \omega t$

$$y = \frac{y_0}{2}(1 - \cot 2\omega t)$$

$$y - \frac{y_0}{2} = -\frac{y_0}{2}\cos 2\omega t$$

$Y = A \cos 2\omega t$

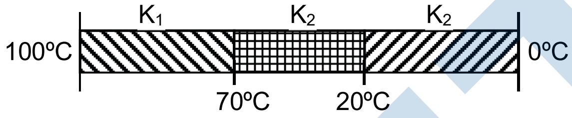
$$2\omega = \sqrt{\frac{k}{m}}$$

maximum displacement $= y_0 = \frac{mg}{k}$

$y_0 \times (2\omega)^2 = 2g$

$$\omega = \sqrt{\frac{g}{2y_0}}$$

7. The rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is :



(1) $K_1 < K_2 < K_3$ (2) $K_1 : K_3 = 2 : 3$ (3) $K_1 : K_2 = 5 : 2$ (4) $K_1 > K_2 > K_3$

Ans. (2)

Sol. $K_1(100 - 70) = K_2(70 - 20) = K_3(20 - 0)$

$K_1 30 = K_2(50) = K_3(20)$

$\Rightarrow K_1 : K_2 : K_3 = 10 : 6 : 15$

$K_1 : K_3 = 2 : 3$

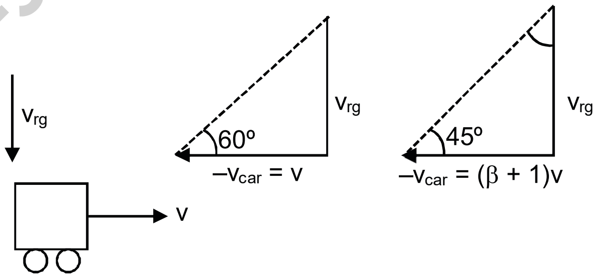
8. When a car is at rest, its driver sees rain drops falling on it vertically. When driving the car with speed v, he sees that rain drops coming at an angle 60° from the horizontal. On further increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45°. The value of β is close to :

(1) 0.41 (2) 0.50 (3) 0.73 (4) 0.37

Ans. (3)

Sol. $\tan 60° = \frac{v_{rg}}{v}$ ....(1)

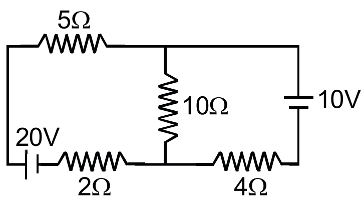
$\tan 45° = \frac{v_{rg}}{(\beta + 1)v}$ ....(2)



$\sqrt{3}v = (\beta + 1)v$

$\beta = \sqrt{3} - 1 = 0.732$

9. In the figure shown, the current in the 10 V battery is close to :



(1) 0.42 A from positive to negative terminal (2) 0.71 A from positive to negative terminal

(3) 0.36 A from negative to positive terminal (4) 0.21 A from positive to negative terminal

Ans. (4)

Sol. $-5i_2 - 10(i_1 + i_2) - 2i_2 + 20 = 0$

$-10i_1 - 17i_2 + 20 = 0$

$-10 + 4i_1 + 10(i_1 + i_2) = 0$

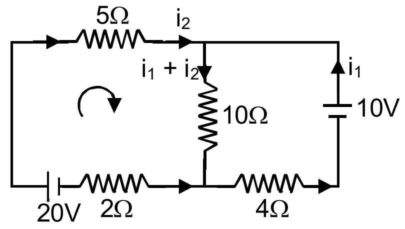
$14i_1 + 10i_2 + 10 = 0$

$10i_1 + 17i_2 = 20 \rightarrow \times 10$

$14i_1 + 10i_2 = 10 \rightarrow 17$

$-138i_1 = 30$

$i_1 = -\frac{30}{138} = -0.217$



i_1 is negative it means current flows from positive to negative terminal

10. A fluid is flowing through a horizontal pipe of varying cross-section, with v ms^{-1} at a point where the pressure is P Pascal. At another point where pressure $\frac{P}{2}$ Pascal its speed is V ms–1 . If the density of the fluid is ρ kg-m^{-3} and the flow is streamline, then V is equal to

(1) $\sqrt{\frac{P}{2\rho} + \upsilon^2}$ (2) $\sqrt{\frac{P}{\rho} + \upsilon^2}$ (3) $\sqrt{\frac{2P}{\rho} + \upsilon^2}$ (4) $\sqrt{\frac{P}{\rho} + \upsilon}$

Ans. (2)

Sol. $P + \frac{1}{2}\rho\upsilon^2 = \frac{P}{2} + \frac{1}{2}\rho V^2$

$\frac{P}{2} + \frac{1}{2}\rho\upsilon^2 = \frac{1}{2}\rho V^2 \quad ; \quad V = \sqrt{\upsilon^2 + \frac{P}{\rho}}$

11. Consider the force F on a charge 'q' due to a uniformly charged spherical shell of radius R carrying charge Q distributed uniformly over it. Which one of the following statements is true for F, if 'q' is placed at distance r from the centre of the shell ?

(1) $F = \frac{1}{4\pi \epsilon_0}\frac{Qq}{r^2}$ for all r (2) $F = \frac{1}{4\pi \epsilon_0}\frac{Qq}{R^2}$ for r < R

(3) $\frac{1}{4\pi \epsilon_0}\frac{Qq}{R^2} > F > 0$ for r < R (4) $F = \frac{1}{4\pi \epsilon_0}\frac{Qq}{r^2}$ for r > R

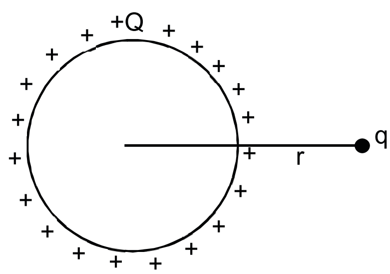
Ans. (4)

Sol. E = O, r < R

$= \frac{qQ}{r^2}, r \geq R$

Inside F = 0 and outside sphere

$F = \frac{2Qq}{4\pi \epsilon_0 r^2}$



12. Assuming the nitrogen molecule is moving with r.m.s. velocity at 400 K, the de-Broglie wavelength of nitrogen molecule is close to : (Given : nitrogen molecule weight : 4.64×10^{-26} kg, Boltzman constant : 1.38×10^{-23} J/K, Planck constant : 6.63×10^{-34} J-s)

(1) 0.34 Å (2) 0.44 Å (3) 0.20 Å (4) 0.24 Å

Ans. (4)

Sol. $\lambda = \frac{h}{\sqrt{2mk}} = \frac{h}{\sqrt{m \times \frac{3}{2}kT}} = \frac{h}{\sqrt{3mkT}}$

$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 4.64 \times 10^{-26} \times 1.38 \times 10^{-13} \times 400}} = 0.24\text{Å}$

13. A particle moving in the xy-plane experiences a velocity dependent force $\vec{F} = k(\upsilon_y \hat{i} + \upsilon_x \hat{j})$, where υ_x and υ_y are the x and y components of its velocity $\vec{\upsilon}$. If $\vec{a}$ is the acceleration of the particle, then which of the following statements is true for the particle ?

(1) quantity $\vec{\upsilon} \cdot \vec{a}$ is constant in time
(2) quantity $\vec{\upsilon} \times \vec{a}$ is constant in time
(3) $\vec{F}$ arises due to a magnetic field
(4) kinetic energy of particle is constant in time

Ans. (2)

Sol. $\vec{F} = k\vec{v}$

$\Rightarrow \vec{F} \,||\, \vec{v}$

$\Rightarrow \vec{a} \,||\, \vec{v}$

$\Rightarrow \vec{v} \times \vec{a} = 0$ always

14. A double convex lens has power P and same radii of curvature r of both the surfaces. The radius of curvature of a surface of a plano–convex lens made of the same material with power 1.5 P is :

(1) $\frac{3R}{2}$ (2) $\frac{R}{2}$ (3) $\frac{R}{3}$ (4) 2R

Ans. (3)

Sol. $\frac{1}{P} = (n-1)\left(\frac{1}{R} - \frac{1}{-R}\right)$

$\frac{1}{P} = (n-1)\frac{2}{R}$ ....(1)

$\frac{1}{1.5P} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$ ....(2)

$\frac{2}{3P} = \frac{n-1}{R}$

From (1) and (2)

$\frac{3}{2} = \frac{R'}{2R}$

$\Rightarrow R' = \frac{R}{3}$

15. Given the masses of various atomic particles m_P = 1,0072 u, m_n = 1,0087 u, m_e = 0.000548 u, $m_{\bar{v}} = 0$, m_d = 2.0141 u, where p = proton, n ≡ neutron, e ≡ electron, $\bar{v}$ ≡ antineutrino and $\bar{d}$ ≡ deuteron. Which of the following process is allowed by momentum and energy conservation :

(1) $n + p \rightarrow d + \gamma$

(2) $p \rightarrow n + e^{+} + \bar{\upsilon}$

(3) n + n → deuterium atom (electron bound to the nucleus)

(4) $e^{+} + e^{-} \rightarrow \gamma$

Ans. (1)

16. The linear mass density of a thin rod AB of length L varies from A to B as $\lambda(x) = \lambda_0\left(1+\frac{x}{L}\right)$, where x is the distance from A. If M is the mass of the rod then its moment of inertia about an axis passing through A and perpendicular to the rod :

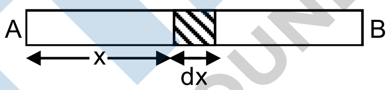
(1) $\frac{5}{12}ML^2$ (2) $\frac{3}{7}ML^2$ (3) $\frac{7}{18}ML^2$ (4) $\frac{2}{5}ML^2$

Ans. (3)

Sol. dm = λ.dv

dI = dm . x^2

$= \lambda_0\left(1+\frac{x}{L}\right)x^2dx$



$I = \lambda_0 \int_0^1 \left(x^2 + \frac{x_3}{L}\right)dx$

$= x_0 \left|\frac{x^3}{3} + \frac{x_4}{4L}\right|_0^L$

$= x_0\left[\frac{L^3}{3} + \frac{L_3}{4}\right]$

$I = \frac{7\lambda_0 L^3}{12}$

$m = \int_0^L \lambda dx = \frac{3\lambda_0 L}{2}$

$\frac{I}{m} = \frac{7L^2}{12 \times \frac{3}{2}} = \frac{7L^2}{18} \Rightarrow I = \frac{7}{18}mL^2$

17. A circuit to verify Ohm's law uses ammeter and voltmeter in series or parallel connected corrected correctly to the resistor. In the circuit :

(1) ammeter is always used in parallel and voltmeter is series

(2) ammeter is always connected in series and voltmeter in parallel

(3) Both ammeter and voltmeter must be connected in parallel

(4) Both ammeter and voltmeter must be connected in series

Ans. (2)

Sol. Theory Based

18. In a dilute gas at pressure P and temperature 't', the mean time between successive collision of a molecule varies with T as :

(1) T (2) $\frac{1}{\sqrt{T}}$ (3) $\sqrt{T}$ (4) $\frac{1}{T}$

Ans. (2)

Sol. $\tau = \frac{\lambda}{v_{rms}}$

$\tau \propto \frac{\lambda}{\sqrt{T}}$

19. Two planets have masses M and 16M and their radii are a and 2a, respectively. The separation between the centres of the planets is 10a. A body of mass m is fired from the surface of the larger planet towards the smaller planet along the line joining their centres. For the body to be able to reach at the surface of smaller planet, the minimum firing speed needed is :

(1) $\frac{3}{2}\sqrt{\frac{5GM}{a}}$ (2) $\sqrt{\frac{GM^2}{ma}}$ (3) $2\sqrt{\frac{GM}{a}}$ (4) $4\sqrt{\frac{GM}{a}}$

Ans. (1)

Sol. $\frac{1}{2}mu^2 + \left[\frac{-G \times 10Mm}{2a} - \frac{GMm}{8a}\right] = 0 - \frac{G \times 16Mm}{8a} - \frac{GM \times m}{2a}$

$\Rightarrow u = \sqrt{\frac{45GM}{4a}}$

$u = \frac{3}{2}\sqrt{\frac{5GM}{a}}$

20. A student measuring the diameter of a pencil of circular cross-section with the help of a vernier scale records the following four readings 5.50 mm, 5.55 mm. 5.34 mm ; 5.65 mm. The average of these four readings is 5.5375 mm and the standard deviation of the data is 0.07395 mm. The average diameter of the pencil should therefore be recorded as :

(1) (5.54 ± 0.07) mm (2) (5.538 ± 0.074) mm

(3) (5.5375 ± 0.0740) mm (4) (5.5375 ± 0.0739) mm

Ans. (1)

Sol. Least count in 0.01 mm therefore option (1) is possible.

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

21. In a series LR circuit, power of 400 W is dissipated from a source of 250 V, 50 Hz. The power factor of the circuit is 0.8. In order to bring the power factor to unity, a capacitor of value C is added in series to the L and R. Taking the value of C as $\left(\frac{n}{3\pi}\right)\mu F$, then value of n is ___________

Ans. (400)

Sol. P = V_m.in.cosϕ

400 = 250 × 1m × 0.8

i_{rms} = 2A

$(1m)^2$. R = P

4 × R = 400 0 ⇒ R = 100Ω.

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$$

$$0.8 = \frac{100}{\sqrt{100^2 + X_L^2}}$$

$$100^2 + X_L^2 = \left(\frac{100}{0.8}\right)^2$$

X_L = 75Ω

Power factor is unity

$X_C = X_L = 75$

$$\frac{1}{\omega C} = 75$$

$$\Rightarrow C = \frac{1}{75 \times 2H \times 50} = \frac{1}{7500\pi}F$$

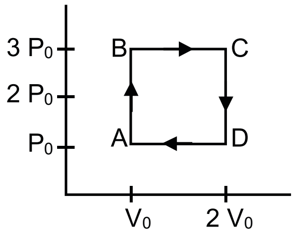
3π × 2500

$$= \frac{1}{3\pi} \times 4 \times 10^2 mF$$

$$= \frac{400}{3\pi}\mu F$$

N = 400

22. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to ______________



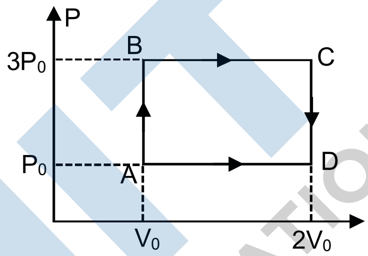
Ans. (19)

Sol. $W = 2P_0v_0$

$Q_+ = W_{AB} + W_{BC}$

$= n.c_v\Delta T_{AB} + nC_P\Delta T_BC$

$$= \frac{3}{2} \times [3P_0V_0 - P_0V_0] + \frac{5}{2}[6P_0V_0 - 3P_0V_0]$$



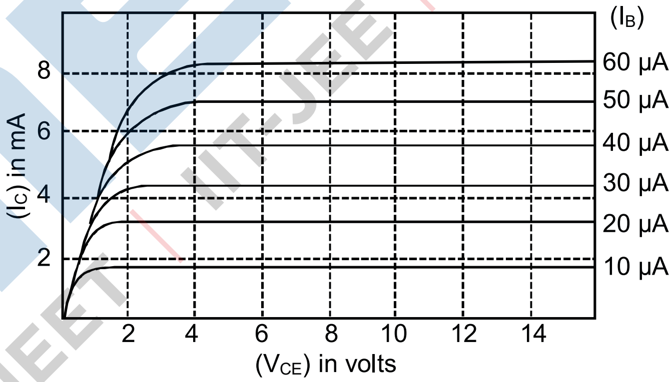
$$= 3P_0V_0 + \frac{5}{2}P_0V_0$$

$$= \frac{21}{2}P_0V_0$$

$$\eta = \frac{W}{Q_+} = \frac{2P_0V_0}{\frac{21}{2}P_0V_0} = \frac{4}{21}$$

$$\eta\% = \frac{400}{21} \approx 19$$

23. The output characteristics of a transistor is shown in the figure. When V_{CE} is 10V and I_C = 4.0 mA, then value of β_{ac} is __________



Ans. (150)

Sol. $$\beta = \left(\frac{\Delta i_C}{\Delta i_B}\right)_{V_{CE}=10} = 150$$

24. A Young's double-slit experiment is performed using monochromatic light of wavelength λ. The intensity of light at a point on the screen, where the path difference is λ, is K units. The intensity of light at a point where the path difference is $\frac{\lambda}{6}$ is given by $\frac{nK}{12}$, where n is an integer. The value of n is __________

Ans. 9

Sol. $I = I_0 \cos^2 \frac{\phi}{2}$

$= k\cos^2\left(\frac{\pi}{\lambda} \times \frac{\lambda}{6}\right)$

$= k\cos^2\left(\frac{\pi}{6}\right)$

$= \frac{3k}{4} = \frac{9k}{12}$

n = 9

25. The centre of mass of a solid hemisphere of radius 8 cm is x cm from the centre of the flat surface. Then value of x is __________

Ans. (3)

Sol. $h_{cm} = \frac{3R}{8} = \frac{3 \times 8}{8} = 3cm$

PART-B : CHEMISTRY

SECTION – 1 : (Maximum Marks : 80)

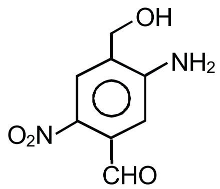
Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

26. The IUPAC name of the following compound is:



(1) 2-nitro-4-hydroxymethyl-5 amino benzaldehyde

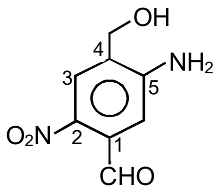
(2) 3-amino-4-hydroxymethyl-5-nitrogenzaldehyde

(3) 4-amino-2-formyl-5-hydroxymethyl nitrogenzene

(4) 5-amino-4-hydroxymethyl-2-nitrobenzaldehyde

Ans. (4)

Sol.



5-Amino-4-(hydroxymethyl)-2-nitro benzene carbaldehyde.

27. Match the following :

Test / Method	Reagent
(i) Lucas Test	(a) $C_6H_5SO_2Cl$/aq. KOH
(ii) Dumas method	(b) $HNO_3/AgNO_3$
(iii) Kjeldahl's method	(c) CuO/CO_3
(iv) Hinsberg Test	(d) Conc. HCl and $ZnCl_2$
	(e) H_2SO_4

(1) (i)-(d), (ii)-(c), (iii)-(b), (iv)-(e)

(2) (i)-(d), (ii)-(c), (iii)-(e), (iv)-(a)

(3) (i)-(b), (ii)-(a), (iii)-(c), (iv)-(d)

(4) (i)-(b), (ii)-(d), (iii)-(e), (iv)-(a)

Ans. (2)

Sol. (I) Lucas reagent $\rightarrow$ Only $ZnCl_2$/Conc.HCl

(II) Dumas method $\rightarrow$ CuO/Δ

(III) Kjeldahl's method $\rightarrow$ Conc. H_2SO_4/Δ

(IV) Heinsberg reagent $\rightarrow$ C_6H_5 SO_2Cl / aq.NaOH

28. For a d^4 metal ion in an octahedral field, the correct electronic configuration is:

(1) $t_{2g}^4 e_g^0$ when $\Delta_0 < P$ (2) $t_{2g}^3 e_g^1$ when $\Delta_0 > P$ (3) $t_{2g}^3 e_g^1$ when $\Delta_0 < P$ (4) $e_g^2 t_{2g}^2$ when $\Delta_0 < P$

Ans. (3)

Sol. For d^4 configuration if $\Delta_0 < P$ the electronic configuration is t_{2g}^3, e_g^1.

29. The element that can be refined by distillation is:

(1) zinc (2) tin (3) gallium (4) nickel

Ans. (1)

Sol. Zn, Cd & Hg are purified by fractional distillation process.

30. Mischmetal is an alloy consisting mainly of:

(1) Lanthanoid metals (2) lanthanoid and actinoid metals

(3) actinoid metal (4) actinoid and transition metals

Ans. (1)

Sol. Misch metal consists of Lanthanide metal (≈95%) and iron (≈5%) and traces of S, C, Ca and Al.

31. Dihydrogen of high purity (>99.95%) is obtained through:

(1) The electrolysis of warm Ba(OH)2 solution using Ni electrodes.

(2) The electrolysis of brine solution.

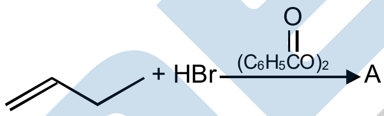
(3) The reaction of Zn with dilute HCl.

(4) The electrolysis of acidified water using Pt electrodes.

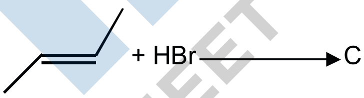
Ans. (1)

Sol. Dihydrogen of high degree of purity (>99.95%) is obtained by the electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

32. The increasing order of the boiling points of the major products A, B and C of the following reactions will be:

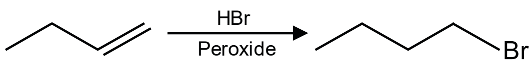
(a) 

(b) 

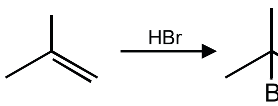
(c) 

(1) A < B < C (2) A < C < B (3) B < C < A (4) C < A < B

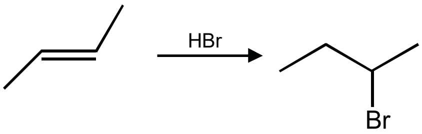
Ans. (3)

Sol. (a) 

(A)

(b) 

(B)

(c)  (C)

The boiling points of isomeric halo alkanes decrease with increase in branching.

33. Match the following compounds (Column-I) with their uses (Column-II):

Column - I	Column - I
(I) $Ca(OH)_2$	(A) casts of statues
(II) NaCl	(B) white wash
(III) $CaSO_4.\frac{1}{2}H_2O$	(C) antacid
(IV) $CaCO_3$	(D) washing soda preparation

(1) (I)-(D), (II)-(A), (III)-(C), (IV)-(B)
(2) (I)-(B), (II)-(D), (III)-(A), (IV)-(C)
(3) (I)-(D), (II)-(D), (III)-(B), (IV)-(A)
(4) (I)-(B), (II)-(C), (III)-(D), (IV)-(A)

Ans. (2)

Sol. (i) $Ca(OH)_2$ is used in white wash.

(ii) Plaster of paris is used in making of molds for plaster statues.

(iii) NaCl is used in preparation of washing soda.

(iv) A suspension of $Mg(OH)_2$ in water is used in medicine as an antacid under name of milk of magnesia.

34. A set of solutions is prepared using 180 g of water as a solvent and 10 g of different non-volatile solutes A, B and c. The relative lowering of vapour pressure in the presence of these solutes are in the order [Given, molar mass of A = 100 g mol^{-1}; B = 200 g mol^{-1}; C = 10,000 g mol^{-1}]

(1) A > B > C (2) B > C > A (3) C > B > A (4) A > C > B

Ans. (1)

Sol. Relative lowering in vapour pressure depends on no. of mole of solute greater the no. of mole of solute greater in RLVP and smaller will be vapour pressure.

So order of vapour pressure is B > C > A.

35. The value of K_C us 64 at 800 K for the reaction

$N_2\,(g) + 3H_2\,(g) \rightarrow 2\,NH_3\,(g)$

The value of K_C for the following reaction is:

$$NH_3(g) \rightarrow \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$$

(1) $\frac{1}{64}$ (2) 8 (3) $\frac{1}{4}$ (4) $\frac{1}{8}$

Ans. (4)

Sol. $N_2(g) + 3H_2 \rightleftharpoons 2NH_3(g)$

$$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3} = 64$$

For the reaction

$$NH_3(g) \rightleftharpoons \frac{1}{2}N_2(g) + \frac{3}{2}H_2(g)$$

$$K'_C = \frac{[N_2]^{1/2}[H_2]^{3/2}}{[NH_3]} = \frac{1}{\sqrt{K_C}} = \frac{1}{\sqrt{64}} = \frac{1}{8}$$

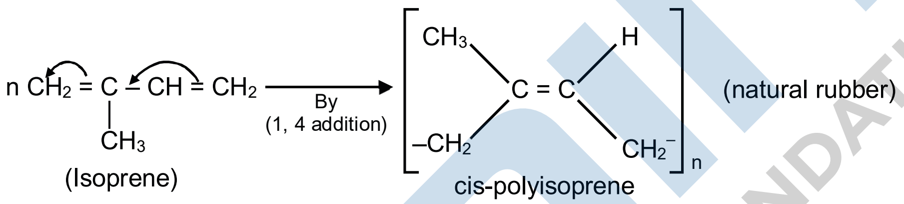
36. The correct match between Item – I and Item – II is:

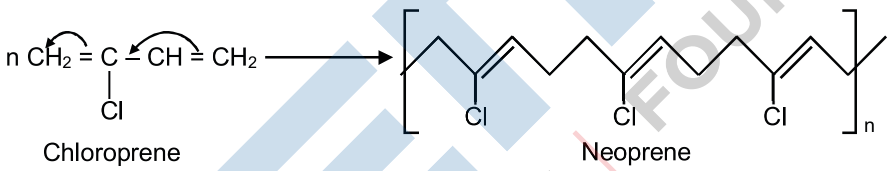
Item – I	Item – II
(a) Natural rubber	(I) 1, 3-butadiene + styrene
(b) Neoprene	(II) 1, 3-butadiene
(c) Buna-N	(III) Chloroprene
(d) Buna-S	(IV) Isoprene

(1) (a)-(IV), (b)-(III), (c)-(II), (d)-(I)
(2) (a)-(III), (b)-(IV), (c)-(II), (d)-(I)
(3) (a)-(IV), (b)-(III), (c)-(I), (d)-(II)
(4) (a)-(III), (b)-(IV), (c)-(I), (d)-(II)

Ans. (1)

Sol. d





$$nCH_2 = CH - CH = CH_2 + nCH_2 = \underset{|}{\overset{CN}{C}}H \xrightarrow{\text{Copoly merisation}} \left(CH_2 - CH = CH - CH_2 - CH_2 - \overset{CN}{\underset{}{C}}H\right)_n$$

1,3 – Butadiene   Acry ionitrile   Buna-N

$$nCH_2 = CH - CH = CH_2 + \underset{C_6H_5}{\underset{|}{CH}} = CH_2 \xrightarrow[\text{Heat}]{\text{Na}} \left(CH_2 - CH = CH - CH_2 - \underset{C_6H_5}{\underset{|}{CH}} - CH_2\right)_n$$

Butadiene   Styrene   Buna-S

37. A crystal is made up of metal ions 'M_1' and 'M_2' and oxide ions. Oxide ions form a ccp lattice structure. The cation 'M_1' occupies 50% of octahedral voids and the cation 'M_2' occupies 12.5% of tetrahedral voids of oxide lattice. The oxidation numbers of 'M_1' and 'M_2' are, respectively:

(1) + 1, +3   (2) +3, +1   (3) +2, +4   (4) +4, +2

Ans. (3)

Sol. In the ccp lattice of oxide ions effective number of O^{-2} ions $= 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

In the ccp lattice,

No. of octahedral voids = 4

No. of tetrahedral voids = 8

Given M_1 atoms occupies 50% of octahedral voids and M_2 atoms occupies 12.5 of tetrahedral voids

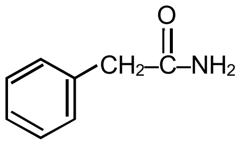
No. of M_1 metal atoms $= 4 \times \frac{50}{100} = 2$

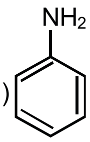
No. of M_2 metal atoms $= 8 \times \frac{12.5}{100} = 1$

$\therefore$ Formula of the compound = $(M_1)_2(M_2)O_4$

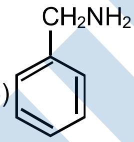
$\therefore$ Oxidation states of metals M_1 & M_2 respectively are +2 and +4.

38. Which of the following compounds can be prepared in good yield by Gabriel phthalimide synthesis?

(1) 

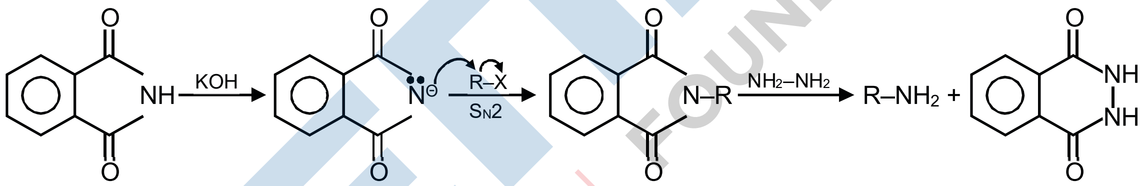
(2) 

(3) $CH_3–CH_2–NHCH_3$

(4) 

Ans. (4)

Sol. From Gabriel phthalimide reaction, 1° Amine can be prepared.



39. Reaction of an inorganic sulphite X with dilute H_2SO_4 generates compound Y. Reaction of Y with NaOH gives X. Further, the reaction of X with Y and water affords compound Z. Y and X, respectively, are:

(1) SO_2 and Na_2SO_3 (2) S and Na_2SO_3 (3) SO_2 and $NaHSO_3$ (4) SO_3 and $NaHSO_3$

Ans. (3)

Sol. $\underset{'X'}{NaHSO_3} + dil.H_2SO_4 \longrightarrow \underset{'Y'}{SO_2} \xrightarrow{NaOH} \underset{'X'}{NaHSO_3} \xrightarrow{dil.H_2SO_4} SO_2$

40. The average molar mass of chlorine is 35.5 g mol^{-1}. The ratio of ^{35}Cl to ^{37}Cl in naturally occurring chlorine is close to:

(1) 3 : 1 (2) 4 : 1 (3) 2 : 1 (4) 1 : 1

Ans. (1)

Sol.

	^{35}Cl	^{37}Cl
Molar ratio	x	1 – x

$M_{avg.} = 35 \times x + 37(1 - x) = 35.5$

$35x + 37 - 37x = 35.5$

$2x = 1.5$

$$x = \frac{3}{4}$$

So, ratio of $^{35}Cl : {}^{37}Cl = 3 : 1$

41. The reaction of NO with N_2O_4 at 250 K gives:

(1) N_2O_5 (2) NO_2 (3) N_2O_3 (4) N_2O

Ans. (3)

Sol. $2NO + N_2O_4 \longrightarrow N_2O_3$

42. For the given cell; $Cu(s)|Cu^{2+}(C_1M)||Cu^{2+}(C_2M)|Cu(s)$

change in Gibbs energy (ΔG) is negative, if:

(1) $C_2 = \frac{C_1}{\sqrt{2}}$ (2) $C_2 = \sqrt{2}C_1$ (3) $C_1 = C_2$ (4) $C_1 = 2\,C_2$

Ans. (2)

Sol. For concentration cell $E^0_{cell} = 0$

Anode : $Cu(s) \longrightarrow Cu^{2+}(aq)_A$

Cathode : $Cu^{2+}(aq)_C \longrightarrow Cu(s)$

Overall : $Cu^{2+}(aq)_C \longrightarrow Cu^{2+}(aq)_A$

As $\Delta G = -nF\,E_{cell}$

If $\Delta G = -ve$ then E_{cell} is positive.

$$E_{cell} = E^0_{cell} - \frac{0.059}{2}\log\frac{C_1}{C_2}$$

$$E_{cell} = \frac{-0.059}{2}\log\frac{C_1}{C_2}$$

$E_{cell} > 0 \Rightarrow C_2 > C_1$

43. Which one of the following statement is not true?

(1) Lactose contains α-glycosidic linkage between C_1 of galactose and C_4 of glucose

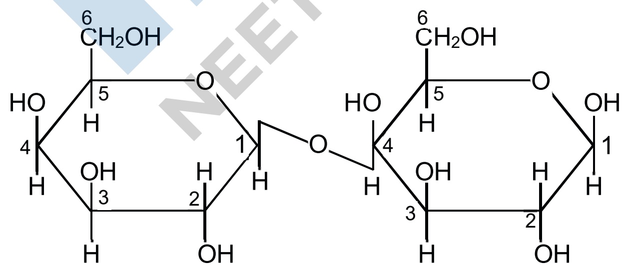
(2) On acid hydrolysis, lactose gives one molecule of D(+)–glucose and one molecule of D(+)-galactose.

(3) Lactose is a reducing sugar and it gives Fehling's test.

(4) Lactose ($C_{11}H_{22}O_{11}$) is a disaccharide and it contains 8 hydroxyl groups.

Ans. (1)

Sol.



The linkage is between C-1 of Galactose and C-4 of Glucose

Lactose (Milk sugar) $\xrightarrow{H_3O^{\oplus}}$ β-galactose + β-glucose

($C_{12}H_{22}O_{11}$)

It is hydrolysed by dilute acids or by the enzyme lactase, to an equimolecular mixture of D(+)-glucose and D(+)-galactose. Lactose is a reducing sugar.

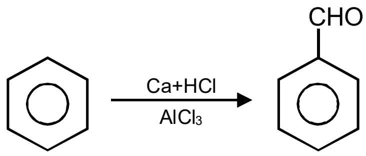
44. The correct match between Item – I (starting material) and Item-II (reagent) for the preparation of benzaldehyde is:

Item-I	Item-II
(I) Benzene	(P) HCl and $SnCl_2$, H_3O^+
(II) Benzonitrile	(Q) H_2, Pd-$BaSO_4$,S and quinoline
(III) Benzoyl Chloride	(R) Co, HCl and $AlCl_3$

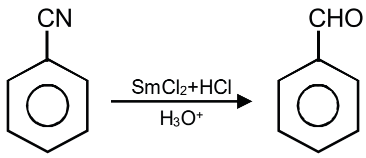
(1) (I)-(R), (II)-(Q) and (III)-(P)
(2) (I)-(Q), (II)-(R) and (III)-(P)
(3) (I)-(P), (II)-(Q) and (III)-(R)
(4) (I)-(R), (II)-(P) and (III)-(Q)

Ans. (4)

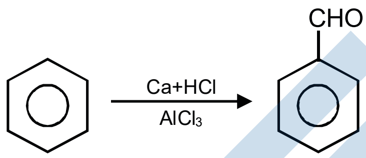
Sol.



Gatterman-Koach reaction



Stephan Reduction



Rosenmund Reduction

45. For a reaction,

$$4\ M(s) + nO_2\ (g) \rightarrow 2\ M_2On(s)$$

The free energy change is plotted as a function of temperature. The temperature below which the oxide is stable could be inferred from the plot as the point at which:

(1) the slope change from positive to zero
(2) the slope changes from negative to positive
(3) The free energy change shows a change from negative to positive value
(4) the slope changes from positive to negative

Ans. (3)

Sol. For oxide to be stable its ΔG value should be negative.

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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46. The atomic number of Unnilunium is______

Ans. (101.00)

Sol. According to IUPAC convention for naming of elements with atomic number more than 100, different digits are written in order and at the end ium is added. For digits following naming is used.

0-nil

1-un

2-bi

3-tri

and so on…

47. The rate of a reaction decreased by 3.555 times when the temperature was changed from 40°C to 30°C. The activation energy (in kJ mol^{-1}) of the reaction is_________

Take; R = 8314 J mol^{-1} K^{-1} In 3.555 = 1.268

Ans. (100.00)

Sol. $$\log\left(\frac{k_2}{k_1}\right) = \frac{Ea}{2.303R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log(3.555) = \frac{Ea}{2.303R}\left[\frac{1}{303} - \frac{1}{313}\right]$$

1.268 × 8.314 × 303 × 313 = 10 Ea

So, Ea = 100 kJ

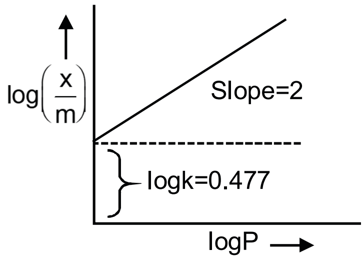
48. For Freundlich adsorption isotherm, a plot of log (x/m) y(-axis) and log p (x-axis) gives a straight line. the intercept and slope for the line is 0.4771 and 2, respectively. The mass of gas, adsorbed per gram of adsorbent if the initial pressure is 0.04 atm is______$\times 10^{-4}$g.

(log 3 = 0.4771)

Ans. (48.00)

Sol. $$\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$$

$$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$$



Slope $= \frac{1}{n} = 2$ So $n = \frac{1}{2}$.

Intercept ⇒ logk = 0.477 So k = Antilog (0.477) = 3

So $\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

$= 3[0.04]^2 = 48 \times 10^{-4}$

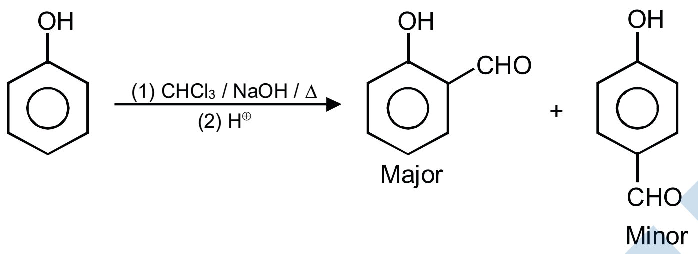
49. A solution of phenol in chloroform when treated with aqueous NaOH gives compound P as a major product. The mass percentage of carbon in P is_______.

(to the nearest integer)

(Atomic mass : C = 12; H = 1; O = 16)

Ans. (69.00)

Sol. Reimer-Tiemann formylation reaction :



50. If the solubility product of AB_2 is 3.20×10^{-11} M^3, then the solubility of AB_2 in pure water is___________ $\times 10^{-4}$ mol L^{-1}

[Assuming that neither kind of ion reacts with water]

Ans. (02.00)

Sol. $AB_2 \rightleftharpoons A^{2+} + 2B^-$

s 2s

$K_{sp} = 4s^3 = 3.20 \times 10^{-11}$

So solubility $= 2 \times 10^{-4}$ mol L^{-1}

PART-C : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Single Choice Type

This section contains 20 Single choice questions. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which Only One is correct.

Full Marks : +4 If ONLY the correct option is chosen.

Negative Marks : –1 (minus one) mark will be deducted for indicating incorrect response.

51. Let $\theta = \frac{\theta}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If B = A + A^4, then det (B) :

(1) is zero (2) is one (3) lies in (2, 3) (4) lies in (1, 2)

Ans. (4)

Sol. $A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow A^4 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$B = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta + \cos\theta & \sin 4\theta + \sin\theta \\ -(\sin 4\theta + \sin\theta) & \cos 4\theta + \cos\theta \end{bmatrix}$

B = $(\cos 4\theta + \cos\theta)^2 + (\sin 4\theta) + \sin\theta)^2$

= 2 + 2 $(\cos 4\theta.\cos\theta + \sin 4\theta.\ \sin\theta)$

= 2 + 2$\cos(4\theta - \theta)$

= 2 + 2.$\cos 3\theta$

$|B| = 2 + 2\cos\frac{3\pi}{5}$

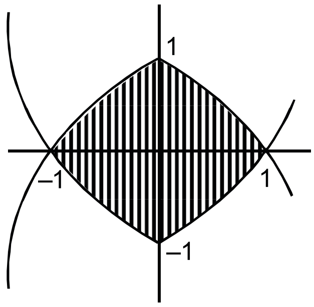
$= 2 - \left(\frac{\sqrt{5}-1}{2}\right) = \frac{5-\sqrt{5}}{2} \in (1,2)$

52. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to :

(1) $\frac{8}{3}$ (2) $\frac{7}{2}$ (3) $\frac{4}{3}$ (4) $\frac{16}{3}$

Ans. (1)

Sol.



Given curves are $y = x^2 - 1$ and $y = 1 - x^2$ so intersection point are (±1, 0)

bounded area $= 4\int_0^1 (1-x^2)dx = 4\left[x - \frac{x^3}{3}\right]_0^1 = 4\left(1-\frac{1}{3}\right) = \frac{8}{3}$ sq. units

53. Let L denote the line in the xy-plane with x and y intercepts as 3 respectively. Then the image of the point (–1, –4) in the line is:

(1) $\left(\frac{8}{5},\frac{29}{5}\right)$ (2) $\left(\frac{11}{5},\frac{28}{5}\right)$ (3) $\left(\frac{29}{5},\frac{8}{5}\right)$ (4) $\left(\frac{29}{5},\frac{11}{5}\right)$

Ans. (2)

Sol. Equation of line is

$$\frac{x}{y}+\frac{y}{1} = 1 \Rightarrow x+3y-3=0$$

If image is (x_1, y_1) then

$$\frac{x_1+1}{1}+\frac{y_1+4}{3} = -2\frac{-1-12-3}{10}$$

$$x_1+1 = \frac{y_1+4}{3} = \frac{16}{5}$$

$$\Rightarrow x_1 = \frac{11}{5}, y_1+1 = \frac{28}{5}$$

54. If α and β are the roots of the equation $2x(2x + 1) = 1$, then β is equal to :

(1) $2\alpha(\alpha - 1)$ (2) $2\alpha(\alpha + 1)$ (3) $2\alpha^2$ (4) $-2\alpha(\alpha + 1)$

Ans. (4)

Sol. Given equation is $2x(2x + 1) = 1 \Rightarrow 4x^2 + 2x - 1 = 0$ ....(1)

roots of equation (1) are α and β

$\therefore$ $\alpha+\beta = -\frac{1}{2} \Rightarrow \beta = -\frac{1}{2}-\alpha$ ....(2)

and

$4\alpha^2+2\alpha-1=0 \Rightarrow \alpha^2 = \frac{1}{4}-\frac{\alpha}{2}$ ....(3)

now

$-2\alpha(\alpha + 1) = -2\alpha^2 - 2\alpha$

$$= -2\left(\frac{1}{4}-\frac{\alpha}{2}\right)-2\alpha = -\frac{1}{2}-\alpha = \beta$$

55. For a suitable chosen real constant a. let a function, $f : R - \{-a\} \to R$ be defined by $f(x) = \frac{a-x}{a+x}$. Further suppose that for any real number $x \neq -a$ and $f(x) \neq -a$, $(fof)(x) = x$. Then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) 3 (2) –3 (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

Ans. (1)

Sol. $fof(x) = \frac{a - f(x)}{a + f(x)} = x$

$\Rightarrow \frac{a - ax}{1+x} = f(x)$

$\Rightarrow \frac{a(1-x)}{1+x} = \frac{a-x}{a+x}$

$\Rightarrow a = 1$

so $f(x) = \frac{1-x}{1+x}$

$f\left(-\frac{1}{2}\right) = 3$

56. If the tangent to the curve, $y = f(x) = x\log_e x$, $(x > 0)$ at a point $(c, f(c))$ is parallel to the line-segment joining the points (1, 0) and (e, e), then c is equal to :

(1) $\frac{e-1}{e}$ (2) $\frac{1}{e-1}$ (3) $e^{\left(\frac{1}{e-1}\right)}$ (4) $e^{\left(\frac{1}{1-e}\right)}$

Ans. (3)

Sol. $f'(c) = 1 + \ell nc = \frac{e}{e-1}$

$\ell nc = \frac{1}{e-1}$

$c = e^{\frac{1}{e-1}}$

57. If the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then |k| equals:

(1) 3 (2) 2 (3) 1 (4) 9

Ans. (1)

Sol. $T_{r+1} = {}^{10}C_r.\left(\frac{-K}{x^2}\right)^r\left(\sqrt{x}\right)^{10-r}$

$= {}^{10}C_r.(-K)^r.\, x^{5-\frac{5r}{2}}$

for constant term $\Rightarrow 5 - \frac{5r}{2} = 0 \Rightarrow r = 2$

$\Rightarrow T_3 = {}^{10}C_2 \,.\, K^2 = 405 \quad \Rightarrow \frac{10(9)}{2}K^2 = 405 \quad \Rightarrow \quad K^2 = 9 \Rightarrow |K| = 3$

58. The probabilities of three events A, B and C are given P(A) = 0.6, P(B) = 0.4 and P(C) = 0.5. If $P(A \cup B) = 0.8$, $P(A \cap C) = 0.3$, $P(A \cap B \cap C) = 0.2$, $P(B \cap C) = \beta$ and $P(A \cup B \cup C) = \alpha$, where $0.85 \le \alpha \le 0.95$, then β lies in the interval :

(1) [0.36, 0.40] (2) [0.35, 0.36] (3) [0.25, 0.35] (4) [0.20, 0.25]

Ans. (3)

Sol. $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$\Rightarrow \alpha = 1.4 - P(A \cap B) - \beta \Rightarrow \alpha + \beta = 1.4 - P(A \cap B)$ …………(1)

again

$P(A \cup B) = P(A) + P(B) - P(A \cap B) \Rightarrow P(A \cap B) = .2$ …………..(2)

by (1) and (2) $\alpha = 1.2 - \beta$

now $0.85 \le \alpha \le 0.95$

$\Rightarrow 0.85 \le 1.2 - \beta \le 0.95 \Rightarrow \beta \in [0.25, 0.35]$

59. The integral $\int_1^2 e^x.x^2(2+\log_e x)dx$ equals :

(1) e(2e – 1) (2) e(4e + 1) (3) $4e^2 - 1$ (4) e(4e – 1)

Ans. (4)

Sol. Let $y = (ex)^x$

$\ell ny = [1 + \ell nx]$

$$\frac{1}{y}\frac{dy}{dx} = (2+\ell n x)$$

$\Rightarrow dy = (ex)^x (2 + \ell nx)\, dx$

$$\int_1^2 e^2.x^2(2+\log_e x)dx = (y)_1^2 = ((ex)^2)_1^2 = 4e^2 - e$$

60. The common difference of the A.P. $b_1, b_2, \ldots, b_m$ is 2 more than common difference of A.P. $a_1, a_2, \ldots, a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to :

(1) 127 (2) 81 (3) – 127 (4) – 81

Ans. (4)

Sol. Let a_1 $a_1 + d$, $a_1 + 2d$ …… first A.P.

$a_{40} = a_1 + 39d = -159$ ……….(1)

$a_{100} = a_1 + 99d = -399$ ……...(2)

from equation (1) and (2)

$d = -4$, $a_1 = -3$

now

$b_{100} = a_{70}$

$\Rightarrow b_1 + 99D = a_1 + 69d$

$b_1 + 99 \times -2 = -3 + 69 \times -4$ (According to question D = d + 2)

$\Rightarrow b_1 = -81$

61. If the normal at an end of latus rectum of an ellipse passes through an extremity of the minor axis, the eccentricity e of the ellipse satisfies:

(1) $e^4 + 2e^2 - 1 = 0$ (2) $e^2 + e - 1 = 0$ (3) $e^2 + 2e - 1 = 0$ (4) $e^4 + e^2 - 1 = 0$

Ans. (4)

Sol. Equation of normal at $\left(ae, \frac{b^2}{a}\right)$

$$\frac{a^2x}{ae} - \frac{b^2y}{b^2/a} = a^2 - b^2$$

It passes through (0, –b)

$ab = a^2e^2$

$a^2b^2 = a^4e^4$ $(b^2 = a^2(1 - e^2))$

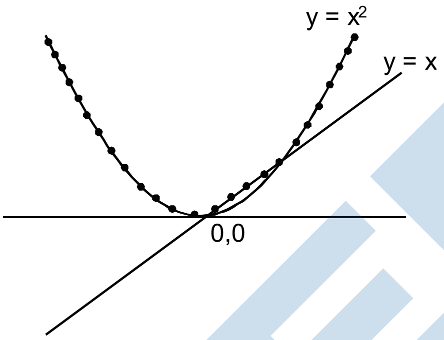
$1 - e^2 = e^4$

62. Let f : R → R be a function defined by $f(x) = \max\{x, x^2\}$. Let S denote the set of all points in R, where f is not differentiable. Then:

(1) ϕ(an empty set) (2) {1} (3) {0} (4) {0, 1}

Ans. (4)

Sol.



63. The set of all real values λ for which the function $f(x) = (1 - \cos^2x).(\lambda + \sin x)$, $x \varepsilon \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, has exactly one maxima and exactly one minima, is :

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$ (2) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (3) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (4) $\left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$

Ans. (4)

Sol. $f(x) = \sin^2x\,(\lambda + \sin x)$

$f'(x) = \sin x \cos x\,(2\lambda + 3\sin x)$

$\sin x = 0$ (one point)

$$\sin x = -\frac{2\lambda}{3} \in (-1, 1) - \{0\} \quad ....(i)$$

$$\lambda \in \left(-\frac{3}{2}, \frac{3}{2}\right) - \{0\}$$

64. Consider the statement: "For an integer n, if $n^3 - 1$ is even, then n is odd". The contrapositive statement of this statement is:

(1) For an integer n, if n is even, then $n^3 - 1$ is odd.

(2) For an integer n, if n is even, then $n^3 - 1$ is even.

(3) For an integer n, if $n^3 - 1$ is not even, then n is not odd.

(4) For an integer n, if n is odd, then $n^3 - 1$ is even.

Ans. (1)

Sol. P : $n^3 - 1$ is even, q : n is odd

contrapositive of $p \to q = \sim q \to \sim p$

$\Rightarrow$ " If n is not odd then $n^3 - 1$ is not even"

$\Rightarrow$ For an integer n, if n is even, then $n^3 - 1$ is odd.

65. The centre of the circle passing through the point (0, 1) and touching the parabola $y = x^2$ at the point (2, 4) is :

(1) $\left(\frac{3}{10}, \frac{16}{5}\right)$ (2) $\left(\frac{-53}{10}, \frac{16}{5}\right)$ (3) $\left(\frac{-16}{5}, \frac{53}{10}\right)$ (4) $\left(\frac{6}{5}, \frac{53}{10}\right)$

Ans. (3)

Sol. $y = x^2$, (2, 4)

tangent at (2, 4) is

$$\frac{1}{2}(y+4) = 2x$$

$y + 4 = 4x \Rightarrow 4x - y - 4 = 0$

Equation of circle $(x - 2)^2 + (y - 4)^2 + \lambda(4x - y - 4) = 0$

it passes through (0, 1)

$\therefore \quad 4 + 9 + \lambda(0 - 1 - 4) = 0$

$$13 = 5\lambda \Rightarrow \lambda = \frac{13}{5}$$

$\therefore$ circle is $x^2 - 4x + 4 + y^2 - 8y + 16 + \frac{13}{5}(4x - y - 4) = 0$

$$\Rightarrow x^2 + y^2 + \left(\frac{52}{5} - 4\right)x - \left(8 + \frac{13}{5}\right)y + 20 - \frac{52}{5} = 0$$

$$\Rightarrow x^2 + y^2 + \frac{32}{5}x - \frac{53}{5}y + \frac{48}{5} = 0$$

$\because$ centre is $\left(-\frac{16}{5}, \frac{53}{10}\right)$

66. If $y = \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec} x$ is the solution of the differential equation, $\frac{dy}{dx} + p(x)y = \frac{2}{\pi}\operatorname{cosec} x, 0 < x < \frac{\pi}{2}$, then the function p(x) is equal to :

(1) tan x (2) cosec x (3) cot x (4) sec x

Ans. (3)

Sol. $y\left(\frac{2}{\pi}x-1\right)\text{cosec}\,x$

$\Rightarrow \frac{dy}{dx}=\frac{2}{\pi}\text{cosec}\,x-\left(\frac{2x}{\pi}-1\right)\text{cosec}\,x\cot x$

$\Rightarrow \frac{dy}{dx}+\left(\frac{2x}{\pi}-1\right)\text{cosec}\,x\cot x=\frac{2}{\pi}\text{cosec}\,x$

$\Rightarrow \frac{dy}{dx}+y\cot x=\frac{2}{\pi}\text{cosec}\,x$

$\Rightarrow$ P(x) = cotx

67. Let z = x + iy be a non-zero complex number such that $z^2 = i\,|z|^2$, where $i=\sqrt{-1}$, then z lies on the:

(1) real axis (2) line, y = x (3) line, y = –x (4) imaginary axis

Ans. (2)

Sol. $(x + iy)^2 = i(x^2 + y^2)$

$\Rightarrow x^2 - y^2 + 2ixy = i\,(x^2 + y^2)$

compare real and imaginary parts

$\Rightarrow$ x = y

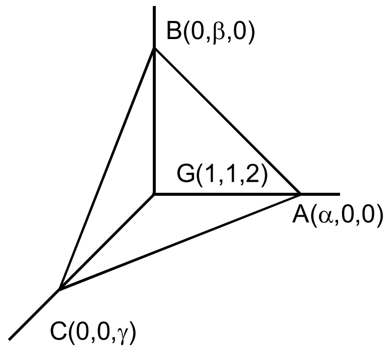
68. A plane P meets the coordinate axes at A, B and C respectively. The centroid of ΔABC is given to be (1,1,2). Then the equation of the line through this centroid and perpendicular to the plane P is:

(1) $\frac{x-1}{2}=\frac{y-1}{1}=\frac{z-2}{1}$ (2) $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$

(3) $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-2}{2}$ (4) $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

Ans. (4)

Sol. Let A(α, 0, 0), B(0, β, 0), C(0, 0, γ) then $G\left(\frac{\alpha}{3},\frac{\beta}{3},\frac{\gamma}{3}\right)\equiv(1,1,2)$



α = 3, β = 3, γ = 6

$\therefore$ equation of plane is $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$

$\Rightarrow \frac{x}{3}+\frac{y}{3}+\frac{z}{6}=1$

$\Rightarrow$ 2x + 2y + z = 6

$\therefore$ required line $\frac{x-1}{2}=\frac{y-1}{2}=\frac{z-2}{1}$

69. The angle of elevation of the summit of a mountain from a point on the ground is 45º. After climbing up one km towards the summit at an inclination of 30º from the ground, the angle of elevation of the summit is found to be 60º. Then the height (in km) of the summit from the ground is:

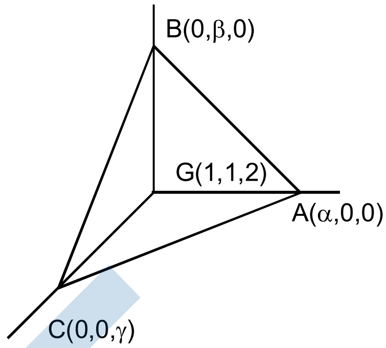
(1) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (2) $\frac{1}{\sqrt{3}+1}$ (3) $\frac{1}{\sqrt{3}-1}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Ans. (3)

Sol. In ΔCDF

$\sin 30° = \frac{z}{1}$ [CD = 1 km(given)]

$z = \frac{1}{2}$ ....(1)



$\cos 30° = \frac{y}{1} \Rightarrow \frac{\sqrt{3}}{2}$

now in ΔABC

$\tan 45° = \frac{h}{x+y}$

$\Rightarrow$ h = x + y

$\Rightarrow$ $x = h - \frac{\sqrt{3}}{2}$ ....(2)

now

In ΔBDE,

$\tan 60° = \frac{h-z}{x}$

$\sqrt{3}x = h - \frac{1}{2} \Rightarrow \sqrt{3}\left(h - \frac{\sqrt{3}}{2}\right) = h - \frac{1}{2} \Rightarrow \left(\sqrt{3}-1\right)h = 1$

$h = \frac{1}{\sqrt{3}-1}$km

70. For all twice differentiable functions $f : R \to R$, with f(0) = f(1) = f'(0) = 0,

(1) f''(x) = 0, for some x ε (0, 1)
(2) f''(x) = 0, at every point x ε (0, 1)
(3) f''(0) = 0
(4) f''(x) ≠ 0, at every point x ε (0, 1)

Ans. (1)

Sol. Applying Rolle's theorem in [0,1] for function f(x)

f'(c) = 0, c ∈ (0,1)

again applying Rolle's theorem in [0,c] for function f'(x)s

$f''(c_1) = 0$, $c_1 \in (0,c)$

option (1) is correct

SECTION – 2 : (Maximum Marks : 20)

This section contains FIVE (05) questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.

If the numerical value has more than two decimal places truncate/round-off the value upto TWO decimal places.

Full Marks : +4 If ONLY the correct option is chosen.

Zero Marks : 0 In all other cases

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71. The number of words (with or without meaning) that can be formed from all the letters of the word "LETTER" in which vowels never come together is……..

Ans. 120

Sol. Consonants are L, T, T, R

Vowels are E,E,

Total number of words (with or without meaning) from letters of word 'LETTER' $= \frac{6!}{2!2!} = 180$

Total number of words (with or without meaning) from letters of word 'LETTER' if vowels are together

$= \frac{5!}{2!} = 60$

$\therefore$ Required = 180 – 60 = 120

72. Consider the data on x taking the values 0, 2, 4, 8, …., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$,…, ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to……..

Ans. (6)

Sol. $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$

x_i (observation)	0	2	2^2		2^n
fi (frequency)	${}^nC_0$	${}^nC_1$	${}^nC_2$		${}^nC_n$

$\frac{0 \times {}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 ....2^n \times {}^nC_n}{{}^nC_0 + {}^nC_1 + {}^nC_2 ..... + {}^nC_n} = \frac{3^n - 1}{2^n} = \frac{728}{2^n}$

$\Rightarrow \quad 3^n = 3^6$

$\Rightarrow \quad n = 6$

73. Suppose that a function f : R $\to$ R satisfies f(x + y) = f(x)f(y) for all x, y ε R and f(1) = 3. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to……..

Ans. (5)

Sol. $f(x) = a^x$

$\Rightarrow \quad f(1) = a = 3$

so $f(x) = 3^x$

$\sum_{i=1}^{n} f(i) = 363$

$\Rightarrow \quad 3 + 3^2 + ... 3^n = 363$

$\frac{3(3^n - 1)}{2} = 363$

$3^n = 243 \Rightarrow n = 5$

74. If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $|\vec{x}+\vec{y}| = |\vec{x}|$ and $2\vec{x}+\lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is.......

Ans. (1)

Sol. $|\vec{x}+\vec{y}|^2 = |\vec{x}|^2$

$\Rightarrow |\vec{y}|^2 + 2\vec{x}\cdot\vec{y} = 0$ ....(1)

and $(2\vec{x}+\lambda\vec{y})\cdot\vec{y} = 0$

$\Rightarrow \lambda|\vec{y}|^2 + 2\vec{x}\cdot\vec{y} = 0$ ....(2)

by (1) and (2) $\lambda = 1$

75. The sum of distinct values of λ for which the system of equations:

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0,$

Has non-zero solutions, is.........

Ans. (3)

Sol. $\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$\begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ 0 & \lambda-3 & -\lambda+3 \\ 3-\lambda & 0 & \lambda-3 \end{vmatrix} = 0$

$C_1 \rightarrow C_1 + C_3$

$\begin{vmatrix} 3\lambda-1 & 3\lambda+1 & 2\lambda \\ 3-\lambda & \lambda-3 & 3-\lambda \\ 0 & 0 & \lambda-3 \end{vmatrix} = 0$

$\Rightarrow \quad (\lambda - 3)^2 [6\lambda] = 0 \Rightarrow \lambda = 0, 3$

sum of values of $\lambda = 3$